Toy Proofs

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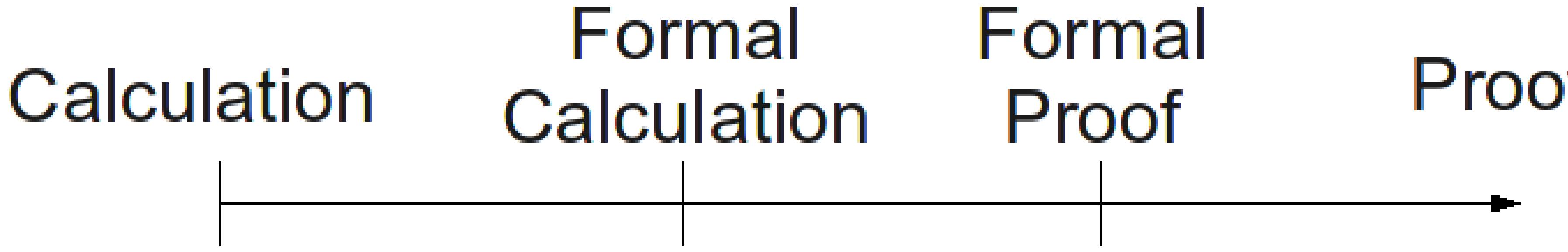
The Problem

How to teach undergraduate students to read and write traditional mathematical proofs?

The Transition

Bridging the Gap



Informal Calculation

$$\begin{aligned}
(x^2 + \sin^3(x))' &= (x^2)' + (\sin^3(x))' \\
&= 2x + (\sin^3(x))' \\
&= 2x + 3\sin^2(x)(\sin(x))' \\
&= 2x + 3\sin^2(x)\cos(x)
\end{aligned}$$

Formal Calculation

$$\begin{aligned}
(x^2 + \sin^3(x))' &= (x^2)' + (\sin^3(x))' && \text{by the sum rule} \\
&= 2x + (\sin^3(x))' && \text{by the power rule} \\
&= 2x + 3\sin^2(x)(\sin(x))' && \text{by the chain rule} \\
&= 2x + 3\sin^2(x)\cos(x) && \text{by the derivative of sin rule}
\end{aligned}$$

Calculation or Proof?

Theorem: $(x^2 + \sin^3(x))' = 2x + 3\sin^2(x)\cos(x)$.

Proof:

$$\begin{aligned}
(x^2 + \sin^3(x))' &= (x^2)' + (\sin^3(x))' && \text{by the sum rule} \\
&= 2x + (\sin^3(x))' && \text{by the power rule} \\
&= 2x + 3\sin^2(x)(\sin(x))' && \text{by the chain rule} \\
&= 2x + 3\sin^2(x)\cos(x) && \text{by the derivative of sin rule.}
\end{aligned}$$

□

Formal Proof to Proof

Proof:

Let $\angle BAC'$ be an angle.

There exists a point C on $\overrightarrow{AC'}$ with $AB \equiv AC$	by the point plotting theorem.
$\triangle ABC$ is isosceles	by the definition of isosceles.
So $\angle ABC \equiv \angle ACB$	by the isosceles triangle theorem.
There exists a midpoint M of segment BC	by SMSG Thm 10.
Hence $BM \equiv MC$	by the definition of midpoint.
Thus $\triangle AMB \equiv \triangle AMC$	by the SAS axiom (S11).

$\vdots$

Proof: Let $\angle BAC'$ be an angle. There exists a point C on $\overrightarrow{AC'}$ with $AB \equiv AC$ by the point plotting theorem. $\triangle ABC$ is isosceles by the definition of isosceles. So $\angle ABC \equiv \angle ACB$ by the isosceles triangle theorem. There exists a midpoint M of segment BC by SMSG Thm 10. Hence $BM \equiv MC$ by the definition of midpoint. Thus $\triangle AMB \equiv \triangle AMC$ by the SAS axiom (S11). So ...

The Formal Proof Game

1. Statements or Expressions (*Toys!*)

The language used in the particular proof or calculation.

2. The Goal (*How to Win!*)

The statement we are trying to prove.

3. Hypotheses (*The Starting Position!*)

Statements we are given initially.

4. Rules of Inference (*Rules of the Game!*)

Allow us to construct new statements from ones we already have.

Proof Parts

Theorem: $(x^2 + \sin^3(x))' = 2x + 3\sin^2(x)\cos(x).$ ← Goal

Proof:

$$\begin{aligned}
(x^2 + \sin^3(x))' &= (x^2)' + (\sin^3(x))' \\
&= 2x + (\sin^3(x))' \\
&= 2x + 3\sin^2(x)(\sin(x))' \\
&= 2x + 3\sin^2(x)\cos(x)
\end{aligned}$$

by the sum rule
by the power rule
by the chain rule
by the derivative of sin rule.

□

Statements

Rules

Toy Proofs

- Introductory Proof-like Games
  - Scrambler
  - TriX
  - Circle-Dot

Circle-Dot

- **Statements**: any sequence of circles and dots
- **Rules** (for any statements W and V)

Axiom 1: ○●

Axiom 2: ●○

Rule 1: Given WV and VW, conclude W

Rule 2: Given W and V, conclude W●V

Rule 3: Given WV●, conclude W○

The Lurch Project

- *Lurch: Software for Teaching Mathematical Proofs*
  National Science Foundation Grant No. 0736644
  - Like a spell-checker or grammar-checker, but for mathematical reasoning.
  - **Mission Statement**: *Lurch should be as indistinguishable from the ordinary activities of mathematics as possible, except for the additional services it provides*.
  - Support for many common undergraduate topics planned: algebra, trigonometry, calculus, logic, set theory, number theory, group theory, etc.

For more info...

The Toy Proof software and these slides are available at the Lurch project home page:

http://lurch.sourceforge.net/