Lurch
Software for teaching mathematical proofs

Nathan Carter, Bentley University
Sage Education Day, December 5, 2009
Why Lurch today?

Those of you here are interested in open source math software for your classroom.

Lurch is not a competitor to Sage; it has a complementary purpose.

The development tools we used (Qt) could build a native version of the Sage notebook.
Lurch Basics

Originally conceived of for upper-division math courses, but will be very flexible.

Developers: Ken Monks (U. Scranton) and me

Support from 2008–2010 from NSF DUE #0736644.

Three-year project (2008–2010), approximately 50% complete

Current release is old
New one coming in a few weeks
Integration Techniques

This worksheet shows how to use Sage to compute definite and indefinite integrals (i.e., with or without bounds). In addition, at the bottom there is code for helping you compute area under curves, using either vertical or horizontal slices.

```python
var('x,y')
(x, y)
```

Using the Integrate command

We need to use the integrate command for all the problems in this unit. The three examples below illustrate

1. How to compute an indefinite integral $\int f(x) \, dx$.
2. How to compute a definite integral $\int_a^b f(x) \, dx$.
3. How to compute an improper integral $\int_a^b f(x) \, dx$ where the function may have an asymptote at $a$ or $b$, or one of the bounds in infinity. Note that if the integral does not converge, Sage will tell you it is divergent.

```python
f(x) = x*sin(x^2)
integrate(f)
    x |--> -1/2*cos(x^2)
```

```python
f(x) = x*sin(x^2)
a=0
b=sqrt(pi/2)
integrate(f,a,b)
    1/2
```
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Compare/Contrast with Sage

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$x \mapsto -\frac{1}{2}\cos(x^2)$

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$\frac{1}{2}$

Explanations, in TeX

Input, to request a computation

Output, the computation's result
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\[ \frac{1}{2} \]
Sage, et al. Computation → Lurch Validation
Purpose:
If your word processor can check your spelling and grammar, why can’t it check your mathematics?
Validation

How much?

Lots

None

Implicit

Explicit

What kind?
Validation

- How much?
  - Lots
  - None

- What kind?
  - Implicit
  - Explicit

- MS Word w/EE, LaTeX, et al.
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LPL

xyAlgebra
Theorem. Suppose \((x \in A) \lor (x \in B)\). Then \(x \in A \cup B\).

Proof:

Proof of \(x \in A \cup B\) goes here.

Given:

\(\Box (x \in A) \lor (x \in B)\)

Goal:

\(\Box x \in A \cup B\)
Theorem. Suppose \((x \in A) \lor (x \in B)\). Then \(x \in A \cup B\).

Proof:

\begin{itemize}
  \item Case 1. \(x \in A\).
  \item Case 2. \(x \in B\).
\end{itemize}

Since this exhausts all the possible cases, we can conclude that \(x \in A \cup B\).
Validation

How much?

Lots vs. None

What kind?

Implicit vs. Explicit

MS Word w/EE, LaTeX, et al.

Proof Designer

LPL xyAlgebra
Validation

How much? | What kind?
---|---
Lots | Proof Designer
None | LPL
Implicit | xyAlgebra
Explicit | MS Word w/EE, LaTeX, et al.

Lurch
Redundant?  No!

Example applets/applications already mentioned are very inflexible/feature-poor.

- Some cannot save, print
- Cannot share data among them
- Cannot extend notation, rules, etc.
Redundant? No!

- Example applets/applications already mentioned are very inflexible/feature-poor.
  - Some cannot save, print
  - Cannot share data among them
  - Cannot extend notation, rules, etc.

- Bringing them under one umbrella means
  - inventing common features only once,
  - building on one another’s strengths, and
  - making the entry barrier lower.
Lurch Demo
Where is Lurch going?

- Next release soon, with several to follow
- Testing in courses in Spring 2010
- Ongoing “topic” building, bug fixes, and feature additions
- More institutions and courses involved in testing and topic development
What kind of topics?

- **Basic Algebra:** Enter any sequence of algebra steps, each validated by an internal CAS.

- **Differential calculus:** Place cursor in a derivative, and select which rule applies.

- **Topic-creation topic:** Insert definitions of symbols, notation conventions, rules, actions, etc. in a readable AND computable way.

- **Whatever anyone builds!** Creating a topic is a good size senior project/directed study work.
Get Involved

lurch.sourceforge.net