The Lurch Project
Validation on Demand

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What is Lurch?
What is Lurch?

- Open Source Software for Validating Mathematics (NSF Grant #0736644)
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- Target Audience: math students and instructors
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- Open Source Software for Validating Mathematics (NSF Grant #0736644)
- Target Audience: math students and instructors
- Completely customizable!
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Mission Statement

Lurch should be as indistinguishable from the ordinary activities of mathematics as possible, except for the additional services it provides. That is, it should only add to your math experience, not change your math experience.
What do we mean by “validating mathematics”? 
Maria is an All American runner. Keenan has been a Siemens finalist.
Why not reasoning checkers?

Maria is an All American runner. Keenan has been a Siemens finalist. By arithmetic, $1+1=3$. I like Lurch and I do not like Lurch.
Why not reasoning checkers?

Maria is an All American runner. Keenan has been a Siemens finalist. By arithmetic, $1+1=3$. I like Lurch and I do not like Lurch.
Project Status: LurchLite

Lurch is a word processor with the ability to check the steps of your work in many areas of mathematics, from calculus to logic.

Mission statement

1. Lurch should be as indistinguishable from the ordinary activities of mathematics as possible, except for the additional services it provides. That is, it should only add to your math experience, not change your math experience.

2. Lurch should provide the software infrastructure the mathematical community needs for validating rigorous mathematics. That is, it should validate mathematical content created by you -- a "spell-checker" for mathematical rigor.

Download Lurch Lite

Lurch open-source software for validating mathematics

lurch.sourceforge.net
Validation: Proofs & Calculations

Lurch validates Rules

Rule
(a theorem, definition, formula, equation, rule of inference, etc.)
Lurch validates Rules

Inputs
(givens, statements, values, etc.)

Rule
(a theorem, definition, formula, equation, rule of inference, etc.)
Validation: Proofs & Calculations

Lurch validates **Rules**

- **Inputs** (givens, statements, values, etc.)
- **Rule** (a theorem, definition, formula, equation, rule of inference, etc.)
- **Outputs** (conclusions, answers, etc.)
Validation: Proofs & Calculations

Lurch validates Rules

Inputs
(givens, statements, values, etc.)

Rule
(a theorem, definition, formula, equation, rule of inference, etc.)

Rule Name
(for giving reasons)

Outputs
(conclusions, answers, etc.)
Validating Rules

When can a Rule be applied?

Inputs → preconditions → Rule → Outputs

Rule Name
Validation Calculations

Example: A Calculation Rule

$$(F+G)' = F' + G'$$
Example: A Calculation Rule

$$(F+G)'=F'+G'$$

*Derivative of a sum*
Validation Calculations

Example: A Calculation Rule

\[(x^2 + \sin(x))' = (F+G)' = F' + G'\]

Derivative of a sum
Example: A Calculation Rule

\[(x^2 + \sin(x))' = (F + G)' = F' + G'\]

Derivative of a sum
Example: A Calculation Rule

\[(x^2 + \sin(x))' = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin(x))\]

Derivative of a sum
Validating a Definition

Example: A definition as a Rule

\[ \text{Def: } \triangle XYZ \text{ is a right angle iff } |\angle XYZ| = 90^\circ \]
Validating a Definition

Example: A definition as a Rule

\[ \triangle ABC \text{ is a right angle} \]

**Def:** \( \triangle XYZ \text{ is a right angle iff } |\angle XYZ|=90^\circ \)

*Definition of Right Angle*
Validating a Definition

Example: A definition as a Rule

\[ \triangle ABC \text{ is a right angle} \Rightarrow |\triangle XYZ| = 90^\circ \]

Definition of Right Angle
Q: What should Lurch do for the user?

$$(x^2 + \sin(x))'$$

**F** = $x^2$, **G** = $\sin(x)$ are functions of one variable

$$(F+G)' = F' + G'$$

**Derivative of a sum**

$$(x^2)' + \sin(x)'$$
Q: What should Lurch do for the user?

Find possible inputs?

\[(x^2 + \sin(x))'\]

F = \(x^2\), G = \(\sin(x)\)

are functions of one variable

\[(F+G)' = F' + G'\]

Derivative of a sum

\[(x^2)' + \sin(x)'\]
Q: What should Lurch do for the user?

Find possible inputs?

Parse the input?

F=x^2, G=sin(x) are functions of one variable

(F+G)'=F'+G'

Derivative of a sum

(x^2+sin(x))'
Q: What should Lurch do for the user?

Find possible inputs?

Parse the input?

Provide a list of Rules?

\((x^2 + \sin(x))'\)

\(F = x^2, G = \sin(x)\)

\((F+G)' = F' + G'\)

Derivative of a sum

\((x^2)' + \sin(x)'\)
Q: What should Lurch do for the user?

Find possible inputs?

Parse the input?

Provide a list of rules?

Apply the rule?

Derivative of a sum

\((x^2+\sin(x))'\)

\(F=x^2, G=\sin(x)\) are functions of one variable

\((F+G)'=F'+G'\)

\((x^2)'+\sin(x)'\)
Q: What should Lurch do for the user?

- Find possible inputs?
- Parse the input?
- Provide a list of Rules?
- Apply the Rule?
- Check the user's answer?

\[ (x^2 + \sin(x))' \]

\[ F = x^2, G = \sin(x) \text{ are functions of one variable} \]

\[ (F+G)' = F' + G' \]

\[ (x^2)' + \sin(x)' \]

Derivative of a sum
Q: What should Lurch do for the user?

- Find possible inputs?
- Parse the input?
- Provide a list of Rules?
- Apply the Rule?
- Check the user's answer?

Derivative of a sum

\[(x^2 + \sin(x))' = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin(x)) = 2x + \cos(x)\]
Q: What should Lurch do for the user?

Derivative of a sum

\[(x^2 + \sin(x))' = (x^2)' + \sin(x)'

Find possible inputs?

Parse the input?

Provide a list of Rules?

Apply the Rule?

Check the user's answer?

Check the user's reason?

Provide a reason?
Q: What should Lurch do for the user?

A: Whatever you want!
Lurch Topics

A list of Rules is called a Topic

Assignment #1 - Derivatives

1. Comment:
This library allows you to guide Lurch Lite through taking derivatives of basic calculus

Derivative Rules
- of a constant
- of a constant power
- of a non-constant power
- of a sum
- of a difference
- of an expression with a constant coefficient
- of sine
- of cosine
- of tangent
- of cotangent
- of secant
- of cosecant
Current LurchLite Topics

Section 1, theorem:

1. o. Axiom A
2. o Axiom B
3. o Rule 1, lines 1 and 2

Section 2, work:

1. o Axiom A
2. o Theorem
3. o.o. Rule 2, lines 2 and 1

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs
Current LurchLite Topics

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs

Section 1, work:

1 \[ P \land Q \]
2 \[ P \quad \land E \ 1 \]
3 \[ P \land Q \rightarrow P \quad \rightarrow I \ 1,2 \]
Current LurchLite Topics

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs

Derivative Rules
- of a constant
- of a constant power
- of a non-constant power
- of a sum
- of a difference
- of an expression with a constant coefficient
- of sine
- of cosine
- of tangent
- of cotangent
- of secant
- of cosecant
- of logarithm (any base)
Current LurchLite Topics

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs

CAS proof:

\[
\frac{x^2 - 1}{x^2 - x - 2} = \frac{(x-1)(x+1)}{(x+1)(x-2)} \quad \text{OK}
\]

\[
= \frac{x-1}{x-2} \quad \text{not sure}
\]
Assignment #3
Problem 1.a. Prove the isosceles triangle theorem.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\triangle ABC$ is a triangle</td>
</tr>
<tr>
<td>2</td>
<td>Assume $\angle B \cong \angle C$</td>
</tr>
<tr>
<td>3</td>
<td>$BC \cong BC$</td>
</tr>
<tr>
<td>4</td>
<td>$\triangle ABC \cong \triangle ACB$</td>
</tr>
</tbody>
</table>

Logic
- Circle-Dot Axiom Game
- TriX Game
- MIU Proof System
- Classic Propositional Logic

Calculus/Algebra
- Derivatives
- CAS
- Boolean algebra

General Purpose
- Word Processing
- Word Processing w/ Scripting
- Line-Numbered proofs
Flavors of Validation

Implicit Validation

Your proof/calculation is valid because you simply cannot make an invalid one.

Validation on Demand

You just type your document as you normally would, reasons optional, and Lurch only asks you for more details if you request a particular claim be validated.
## Coming Attractions

<table>
<thead>
<tr>
<th>Feature</th>
<th>Current status</th>
<th>Coming soon...</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical formatting</strong></td>
<td>LaTeX formatting via jsMath in limited use in two topics only</td>
<td>LaTeX formatting via jsMath ubiquitous in all topics</td>
</tr>
<tr>
<td><strong>Entering Mathematics</strong></td>
<td>Line numbered proofs requires LaTeX source code. CAS topic uses calculator notation.</td>
<td>LaTeX source, calculator notation, toolbars, and other custom math parsers</td>
</tr>
<tr>
<td><strong>Other formatting</strong></td>
<td>Bold, italics, justifications, and common headers available via toolbars</td>
<td>Arbitrary html and LaTeX formatting available</td>
</tr>
<tr>
<td><strong>Topics</strong></td>
<td>Nine topics available in the current release</td>
<td>Topics in Boolean algebra, predicate logic, set theory, and many, many, more planned</td>
</tr>
<tr>
<td><strong>Developer interface</strong></td>
<td>New topics developed and modified via javascript programming</td>
<td>Instructors can define new topics using standard mathematics notation</td>
</tr>
</tbody>
</table>

**Lurch** open-source software for validating mathematics

[SourceForge](lurch.sourceforge.net)
For more info...

The LurchLite software and these slides are available at the Lurch project home page:

lurch.sourceforge.net